

On the perturbation of domain wall coupled to gravitational waves

Akihiro ISHIBASHI and Hideki ISHIHARA

*Department of Physics, Tokyo Institute of Technology,
Oh-Okayama Meguro-ku, Tokyo 152, Japan*

The equation of motion for domain wall coupled to gravitational field is derived. The domain wall is treated as a source of gravitational field around the wall. The perturbed equation is also obtained with taking account of the gravitational back reaction on the motion of the domain wall.

As a possible evidence of cosmological phase transitions in the early universe, topological defects may remain somewhere in our universe¹. It has been of interest to investigate their behaviors, especially, those accompanied with radiations in cosmological and astrophysical contexts. We shall comment on the equation of motion (EOM) for domain wall coupled to gravity, especially the perturbed EOM for domain wall coupled to gravitational waves.

In the treatment of domain walls, we often apply the thin wall approximation. Then the history of the domain wall is regarded as a 3-dimensional timelike hypersurface (Σ, γ_{ij}) in 4-dimensional spacetime $(M, g_{\mu\nu})$ embedded by $x^\mu = x^\mu(\zeta^j)$, where the 3-dimensional metric is induced by $\gamma_{ij} := g_{\mu\nu}(\partial x^\mu / \partial \zeta^i)(\partial x^\nu / \partial \zeta^j)$. The EOM for the domain wall is derived from the Nambu-Goto action

$$S = -\sigma \int_{\Sigma} d^3\zeta \sqrt{-\gamma} + \rho \int_{M_{inside}} d^4x \sqrt{-g}, \quad (1)$$

where σ is the surface energy density of the wall and ρ is the difference of the vacuum energy densities between the inside and the outside regions of the wall. Taking the variation in the wall position x^μ gives the EOM, $K = -\rho/\sigma$, where K is the trace of the extrinsic curvature K_{ij} of Σ .

The perturbation of the domain wall is described by $\tilde{x}^\mu = x^\mu + \phi n^\mu$, where n^μ is the normal vector to Σ . The scalar field ϕ living on Σ obeys the Klein-Gordon equation,

$$(\square_3 - m^2) \phi = 0, \quad (2)$$

on the background world sheet of the wall^{2,3}. Here \square_3 is the d'Alembertian for γ_{ij} and

$$m^2 := -R_{\mu\nu}q^{\mu\nu} + {}^{(3)}R - \left(\frac{\rho}{\sigma}\right)^2, \quad (3)$$

where $R_{\mu\nu}$ is the spacetime Ricci tensor evaluated at the world hypersheet and ${}^{(3)}R$ the 3-dimensional scalar curvature of (Σ, γ_{ij}) . From Eq. (2), one intuitively expect that the wave propagation of the scalar field represents the oscillation of the deformed wall and such wall oscillations generate gravitational waves.

In the derivation of Eq. (2) one naively assumed that the gravitational back reaction on the wall motion was higher order and hence the influence of gravitational waves could be negligible at least in the first order of wall perturbation.

However, recent investigations of the perturbations of O(3,1) symmetric domain walls^{5,6} demonstrate that the naive assumption noted above is incorrect. In these

works^{5,6}, by the metric junction formalism, the moving domain wall is treated as a source of the gravitational field. It turns out that the perturbative motion of the wall is completely accompanied with the gravitational perturbations and the domain wall oscillates only while incident gravitational waves go across; the domain wall loses its own dynamical degree of freedom. Thus, the gravitational back reaction on a domain wall motion cannot be ignored even in the first order of the perturbation.

Motivated by these works, we shall consider how to derive the perturbed EOM for domain wall coupled to gravitational waves from the Nambu-Goto action with the action for the gravitational field $\int d^4x \sqrt{-g}R/16\pi G$.

Note that, once the wall Σ is regarded as a source of a gravity, Σ is a singular hypersurface and the spacetime metric $g_{\mu\nu}$ is no longer smooth there. Thus, while the intrinsic quantities are continuous and well-defined at Σ , the derivatives of the spacetime metric such as the extrinsic curvature K_{ij} and the spacetime Ricci tensor $R_{\mu\nu}$ are in general discontinuous at Σ and ill-defined on Σ . The EOM therefore has ambiguities in the evaluations of the discontinuous quantities at Σ .

To resolve this problem, we evaluate the discontinuous quantities on Σ to be consistent with the Einstein equations on the whole spacetime $(M, g_{\mu\nu})$. Since the (i, j) components of the Einstein tensor $G^i{}_j$ contains $\partial_\chi K^i{}_j$, where χ is the axis normal to Σ , and the energy momentum tensor $T^\mu{}_\nu$ contains the delta function, one obtain $K^i{}_j|_\Sigma := \lim_{\epsilon \rightarrow 0} \int_{-\epsilon}^{\epsilon} d\chi K^i{}_j \delta(\chi) = \frac{1}{2} \lim_{\epsilon \rightarrow 0} \{ K^i{}_j|_{+\epsilon} + K^i{}_j|_{-\epsilon} \} =: \overline{K}^i{}_j$. Similarly, for R_{ij} , we should consider the averaged one \overline{R}_{ij} . This is indeed what the metric junction formalism yields. Then we obtain the EOM for a domain wall coupled to gravitational field

$$\overline{K} = -\rho/\sigma. \quad (4)$$

Now let us consider the perturbed equation. To study a coupled system of a domain wall and gravitational field, in addition to the wall perturbation ϕ , we further take the metric perturbations $\delta g_{\mu\nu} =: h_{\mu\nu}$ into account. Then we see that the spacetime metric transforms as $g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(x) - 2\phi K_{\mu\nu}(x) + h_{\mu\nu}(x)$. Taking the average for the discontinuous quantities, we obtain the perturbed EOM for the domain wall coupled to gravitational perturbations in the form

$$(\square_3 - \overline{m^2}) \phi(\zeta^i) + J = 0, \quad (5)$$

where

$$J := {}^3D^j h_{\chi j} - \frac{1}{2} \overline{\partial_\chi h^j}_j + \frac{1}{2} (\rho/\sigma) h_{\chi\chi}. \quad (6)$$

In particular spherically symmetric background case, introducing the gauge-invariant variables for the metric perturbations following Gerlach and Sengupta⁷, we can express Eq. (5) in terms of them as

$$(\square_3 - \overline{m^2}) \Xi + J = 0, \quad (7)$$

where

$$\Xi := \phi + X \quad (8)$$

is the gauge-invariant wall displacement variable and X is a quantity corresponding to $n^C P_C$ introduced by Gerlach and Sengupta⁷. For the detail of the derivation, see the paper⁸.

1. A. Vilenkin and E. P. S. Shellard, *Cosmic strings and other topological defects*, (Cambridge University Press, Cambridge, 1994).
2. J. Garriga and A. Vilenkin, *Phys. Rev. D* **44**, 1007 (1991).
3. J. Guven, *Phys. Rev. D* **48**, 4604 (1993).
4. W. Israel, *Nuovo Cimento B* **44**, 1 (1966).
5. H. Kodama, H. Ishihara, and Y. Fujiwara, *Phys. Rev. D* **50**, 7292 (1994).
6. A. Ishibashi and H. Ishihara, *Phys. Rev. D* **56**, 3446 (1997).
7. U. H. Gerlach and U. K. Sengupta, *J. Math. Phys.* **20**, 2540 (1979).
8. A. Ishibashi and H. Ishihara (in preparation).